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# AN EPQ MODEL WITH VARYING RATE OF DETERIORATION AND MIXED DEMAND PATTERN

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***ABSTRACT***

***This paper derives the optimal cycle time and total cost of an EPQ model with a variable rate of deterioration and mixed demand pattern. Demand is inventory dependent during the inventory buildup time and constant demand has been used for during an inventory depletion period. The rate of deterioration is changing after particular time period. A numerical example has been provided to validate the model. The result of sensitivity analysis indicated that total cycle time and cost are sensitive to production rate, increase in deterioration rate and demand rate.***

***KEYWORDS: EPQ, Mixed Demand Pattern & Varying Rates of Deterioration***

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**Original Article**

## 1. INTRODUCTION

In the manufacturing sector, when items are produced internally instead of being obtained from an outside supplier, the EPQ model is often employed to determine the optimal production lot-size that minimizes overall production/inventory costs. The complexity of a resulting EPQ model depends on the assumptions one makes about various parameters of the inventory system. Among these features are demand types, actions for excess demand, production rates, treatment for imperfect quality items, minimal service level constraint, various cost parameters, and issuing policies when items are perishable. The classic EPQ model assumes that manufacturing facility functions perfectly during a production run. However, due to process deterioration or other factors, the generation of imperfect quality items is inevitable.

From product life cycle, demand rate will be constant only when the product maturity is on stage. In the competitive scenario, higher stock of items attracts the customers to purchase more (Teng *et.al* 2005). Deterioration is the common phenomenon in the real production system of products like pharmaceuticals, food, flowers, vegetables, etc. The effect of an imperfect production process, by assuming different rates of deterioration during the production process, on the optimal cycle time had been studied by (Rosenblatt and Lee, 1986). The EOQ model with power form of stock dependent demand for deteriorating items (Teng, *et.al* 2005), investigated that the shape parameter of the demand and the selling price is highly sensitive to optimal solution. The impact of the random machine failure on the EPQ model (Teng, *et.al, 2005*) was that, the demand function and purchase cost is positive and fluctuating with time. The production process can shift at random from, in control to out of control state, during the production process. In such situation, the items produced in out of control state deteriorate at a higher rate, than the normal rate; hence get consumed by demand under the LIFO policy (Garry and Dah, 2006). Behrouz and Babak, developed economic production quantity (EPQ) model, by considering both the depreciation cost of stored

items and process quality cost. Depreciation cost and process quality cost was assumed to be continuous functions of holding time and of production run length. Yuan et.al (2011) derived the optimal manufacturing batch size and number of shipments with scrap, using mathematical modeling and algebraic approach. Ata (2011) discussed EPQ model for multi products single machine, with discrete delivery. Taleizadeh, Naja and Naiki, focused on EPQ model with a production capacity limitation and a random defective production rate. Gede and Hui (2010), analyzed an EPQ model for deteriorating items with stochastic machine unavailability and price- dependent demand. Jinn (2007), used time varying demand and cost, to analyze EPQ model and characterize the influences of both demand and cost, over the length of production run time and the economic production quantity. Hezari (2008) developed EPQ model, by considering imperfect and defective items. The holding cost for defective items has not been considered. David et. al (2009), considered partial backordering with constant demand, to study inventory model. Jiang Tao (2009) considers a multi-item inventory system, where the vendor provides the retailer with delay in payment. Yao (2007) established a model for deteriorating items under delay in payment, in which the demand is a negative exponential function of price. Jia (2009) reported a supply chain system with trade credit. Liao [6] established an EPQ model for deteriorating items with delay in payment. Ardak *et. al.* (2017) developed EPQ model by considering mixed demand pattern; inventory dependent demand at inventory buildup time and constant demand during depletion period. Ardak & Borade (2017), considered time dependent holding cost to developed EPQ model.

Most of the inventory models considered various demand patterns like stock dependent demand, power form demand, ramp type of demand, time dependent demand, selling price dependent demand and exponential demand. It is well known that, the demand rate varies with change in inventory level. From the literature review, it is observed that the different demand pattern has not been discussed by any researcher so far on different time period. The present study may be significant in filling this gap since it aims to develop EPQ model by assuming demand as inventory dependent criteria during the inventory buildup time and constant during an inventory depletion period. This paper has five sections. Research motivation and literature is narrated in section 1. Section 2 contains notations and assumption. The methodology used to develop the model is discussed in section 3. Numerical and sensitivity analysis is discussed in section 4 and finally concluded in section 5

## 2. ASSUMPTION AND NOTATIONS

Following assumption and notations are used to develop the model.

### 2.1 Assumptions

The following assumptions are made in the development of the model.

- The production rate is known and is constant.
- The production rate is greater than the demand.
- The demand is inventory level dependent in uptime and constant in down time.
- Deterioration of the items is varying.
- Inventory holding cost is known and termed as constant.
- Deterioration of the items starts as it enters into inventory.

- Shortages are not allowed.
- Every produced item needs inspection.

**2.2 Notation**

$I_1$ – Inventory level during production uptime, $I_2$ – Inventory level during production down time, $T_1$– Production up time. Maximum inventory buildup in this time, $T_2$–Production down time., $T_2$–Production down time where rate of deterioration changes, P – Rate of production, D – Basic demand, θ – Basic rate of Deterioration, β- Increase in rate of deterioration, (– Inventory dependent consumption rate parameter., H – Holding cost per unit, $C_i$ _ Inspection cost per unit, T – Production cycle time., TC – Total cost., IC – Total Inspection cost, TCT – Total cost per unit time, TI – Total Inventory.

## 3. MODEL FORMULATION

This EPQ model considered inventory dependent demand and constant holding cost and different rate of deteriorating items. The production starts at time zero and inventory start to build up at the rate of P – D, which is the production rate minus consumption rate with deterioration up to time $T_1$ where the production process stops and the inventory on hand reaches its maximum level. After$T_1$, the inventory consumed under the action of demand and deterioration up to time $T_2$. After time $T_2$, the inventory level decreases with the constant consumption rate, D, but the rate of deterioration increases until it becomes zero at times. The objective of this part of this research is to find the optimal inventory cycle time.

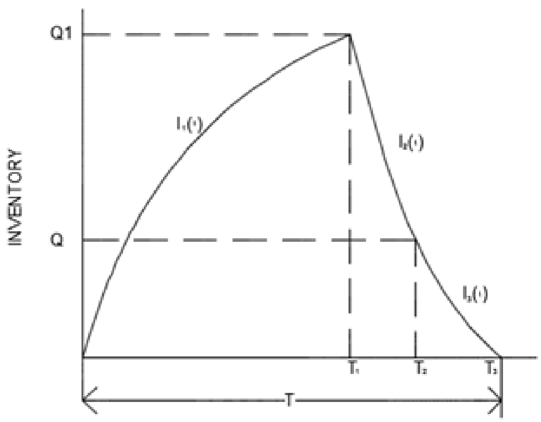


**Figure 1: Inventory System**

As shown in figure.1, the production will start at t = 0, during the time period (0, $T_1$) the inventory will gradually build up with constant deterioration. Later, production stops and build up inventory is consumed to fulfill the demand. For the time period (0, $T_2$), inventory will consumed under the action of constant demand and basic rate of deterioration. Maximum inventory will be at time t = $T_1$. For the time period (0, $T_3$), inventory gets consumed under the action of constant demand and increase rate of deterioration. Production system can be described by the following differential equations.

$$\frac{dI_1(t)}{dt} = P - D - \alpha I_1(t) - \theta I_1(t) \quad 0 \le t \le T_1 \tag{1}$$

During the time interval (0, $T_2$), deterioration of items starts at basic rate. Hence the system is affected by the combined effect of constant demand and deterioration. Hence, the change in inventory level is governed by the following differential equation.

$$\frac{dI_2(t)}{dt} = -D - \theta I_2(t) \quad 0 \le t \le T_2 \tag{2}$$

In a time interval (0, $T_3$), demand rate is constant and the rate of deterioration of items is at increase rate. Hence, the change in inventory level is governed by the following differential equation.

$$\frac{dI_3(t)}{dt} = -D - \beta I_3(t) \quad 0 \le t \le T_3 \tag{3}$$

Initial boundary conditions associated with this equation are, at t = 0, $I_1(t) = 0$, at $t = T_2$, $I_2(T_2) = Q$ and at $t = T_3$, $I_3(T_3) = 0$ the solution for above equations is as follows. These three equations are used in the derivation of our model.

$$I_1(t) = \frac{P-D}{\alpha+\theta}\left[1 - e^{-(\alpha+\theta)t}\right] \tag{4}$$

$$I_2(t) = \frac{-D}{\theta} + \left[Q + \frac{D}{\theta}\right] e^{(\theta)(T_2 - t)} \tag{5}$$

$$I_3(t) = \frac{D}{\theta}\left[e^{\beta(T_3 - t)} - 1\right]. \tag{6}$$

By using initial boundary conditions $I_3(0) = Q$ for equation 6

$$Q = \frac{D}{\beta}\left[e^{\beta(T_3)} - 1\right] = D\left[T_3 + \frac{\beta T_3^2}{2}\right] \tag{7}$$

By using boundary conditions $I_1(T_1) = I_2(0)$ and Eq.7 for equation 4 and 5,

$$\frac{P-D}{\alpha+\theta}\left[1 - e^{-(\alpha+\theta)T_1}\right] = \frac{-D}{\theta} + \left[Q + \frac{D}{\theta}\right] e^{(\theta)(T_2)} \quad T_2 = \frac{(P - D \times T_1)}{D} - T_3 - \frac{\beta T_3^2}{2} \tag{8}$$

Total Inventory is given by

$$TI = \int_0^{T_1} I_1(t)\,dt + \int_0^{T_2} I_2(t)\,dt + \int_0^{T_3} I_3(t)\,dt \tag{9}$$

Total Inventory holding cost is given by

$$IH = H\left[\int_0^{T_1} I_1(t)dt + \int_0^{T_2} I_2(t)dt + \int_0^{T_3} I_3(t)dt\right] = H\left(\frac{P-D}{2} \times T_1^2 + D\left[T_3 + \frac{\beta T_3^2}{2}\right]\left(\frac{(P - D \times T_1)}{D} - T_3 - \frac{\beta T_3^2}{2}\right) + \frac{D}{2} \times T_3^2\right) \tag{10}$$

All produced items need an inspection, so inspection cost is given by,

$$IC = C_i \int_0^{T_1} I_1(t)dt \ = C_i\left(\frac{P-D}{2}\times T_1^2\right) \tag{11}$$

From equations 10 and 11

Total cost = Set up cost + Holding cost + Inspection cost.

$$TC = A + IH + IC \ = A + H\left(\frac{P-D}{2}\times T_1^2 + D\left[T_3 + \frac{\beta T_3^2}{2}\right]\left(\frac{(P-D\times T_1)}{D} - T_3 - \frac{\beta T_3^2}{2}\right) + \frac{D}{2}\times T_3^2\right) + C_i\left(\frac{P-D}{2}\times T_1^2\right) \tag{12}$$

Production cycle time = $T = T_1 + T_2 + T_3$ (13)

$$T = T_1 + \frac{(P-D\times T_1)}{D} - T_3 - \frac{\beta T_3^2}{2} + T_3 \ = T_1\left(1+\frac{P-D}{D}\right) - \frac{\beta T_3^2}{2} \tag{14}$$

Total cost per unit time is given by

$$TCT = \frac{TC}{T} \ = \frac{A + KT_1^2 + K_1T_1T_3 - T_3^2\left[K_2 + K_3 + K_4T_1\right] - K_3T_3^3 - K_5T_3^4}{T_1K_6 - \frac{\beta}{2}T_3^2} \tag{15}$$

Where,

$$K = (H + C_i)\frac{P-D}{2}, K_1 = H(P-D), K_2 = \frac{HD}{2}, K_3 = \frac{HD\beta}{2}, K_4 = \frac{H\beta(P-D)}{2}, K_5 = \frac{HD\beta^2}{4}, K_6 = 1 + + \frac{P-D}{D}$$

Total cost per unit time is a function of $T_1$ and $T_3$. The optimum production up time can be derived from satisfying the equation.

and $\frac{dTCT}{dT_1} = 0$ 16

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By solving equation No. 15 and 16 simultaneously, optimum value of $T_1$ and $T_3$ can be obtained.

$$\frac{dTCT}{dT_3} = \frac{d}{dT_3}\left[\frac{A + KT_1^2 + K_1T_1T_3 - T_3^2\left[K_2 + K_3 + K_4T_1\right] - K_3T_3^3 - K_5T_3^4}{T_1K_6 - \frac{\beta}{2}T_3^2}\right] = 0$$

$$\frac{dTCT}{dT_1} = \frac{d}{dT_1}\left[\frac{A + KT_1^2 + K_1T_1T_3 - T_3^2\left[K_2 + K_3 + K_4T_1\right] - K_3T_3^3 - K_5T_3^4}{T_1K_6 - \frac{\beta}{2}T_3^2}\right] = 0$$

Ignoring higher power of $T_3$ & $T_1$ and after some approximation, following quadratic equation can be obtained.

$$T_3 = \sqrt{\frac{AK_6}{K_2K_6 + K_3K_6 - \frac{\beta}{2}K}} \quad (17)$$

$$T_1^2\left(K_1K_6\right) - T_1\left[2K_2K_6T_3 + 2K_3K_6T_3 + K\beta T_3\right] - A\beta T_3 = 0 \quad (18)$$

Closed form solution can be obtained by considering positive root of equation 18.

## 4. NUMERICAL AND SENSITIVITY ANALYSIS

Numerical example and sensitivity analysis have been carried out to validate the theoretical aspects. The numerical data is adapted from Ardak and Borade. (1) Let, A= Rs.30 per production cycle, P = 2500 units per unit time, D = 1200 units per unit time, (= 0.5, θ = 0.1, a= 2 b = 1.5. And β = 0.2 the optimum value of $T_1$ can be found. The optimum value of $T_1$ is 1.4. The optimum total cost per unit time is TCT = Rs.2411. Sensitivity analysis is carried out by changing each parameter by –40% to +40%, taking one parameter at a time and keeping the others unchanged.

**Table 1: Sensitivity Analysis of $T_1$ and TCT**

| Parameters | Parameter Changes | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | -40% | | -20% | | 20% | | 40% | |
| | $T_1$ | TCT | $T_1$ | TCT | $T_1$ | TCT | $T_1$ | TCT |
| P | 4.42 | 1794.77 | 1.99 | 1986.96 | 1.1 | 3015.83 | 0.96 | 3767.80 |
| D | 1.06 | 3745.65 | 1.21 | 2841.55 | 1.7 | 2236.12 | 2.14 | 2209.72 |
| β | 1.34 | 2330.63 | 1.37 | 2371.09 | 1.4 | 2450.42 | 1.46 | 2489.23 |
| H | 1.86 | 2209.04 | 1.58 | 2301.22 | 1.3 | 2525.52 | 1.17 | 2640.02 |
| Ci | 1.38 | 2155.99 | 1.39 | 2282.64 | 1.4 | 2541.15 | 1.42 | 2673.05 |

A careful study of the computational results as shown in Tables 5.5.6, and within the range of values of the chosen parameters, reveals the following observations:

From Table 1, a higher value of results in higher values of demand rate, β and inspection cost, but lower values of results in higher values of production rate and holding cost. This implies that increase in the demand rate will result in the increase in the length of inventory buildup time. This is expected since if demand rate is higher, the stock will take more time to build up. Increase in deterioration rate (β) will result in an increase in the optimal length of time with positive inventory. This attracts the attention of the inventory managers. With the deterioration rate higher, however, increases inventory buildup time. Increase in production rate results in decrease production up time. During production up time demand used is inventory dependent, so, as rate of production increases inventory increases, but with an increase in inventory demand also increases. The nature of curve for production rate shown in figure 1 indicates this change. The effects of stock dependent demand rate and deterioration on the optimal replenishment policy are significant, and hence should not be ignored in developing inventory models. It is assumed that all produced items needs inspection. Production up time is increasing function of inspection cost. The inspection cost is slightly sensitive to production up time. Figure 2 shows the sensitivity analysis of total cost per unit time. Production rate, demand rate is highly sensitive to TCT. The rate of deterioration and holding cost is moderately sensitive to the total cost per unit time. Increase in demand reduces the cost and increase in production rate increases the total cost. Deterioration rate also increases the TCT.

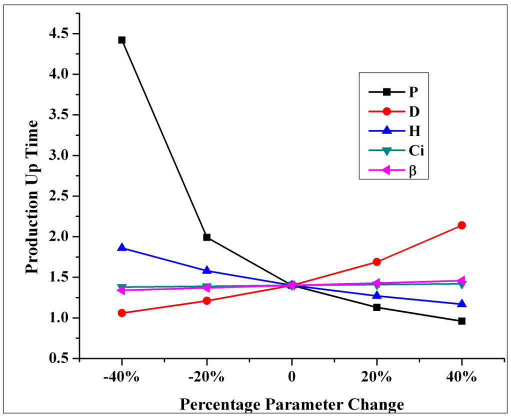


**Figure.2: $T_1$ v/s Parameter Change**

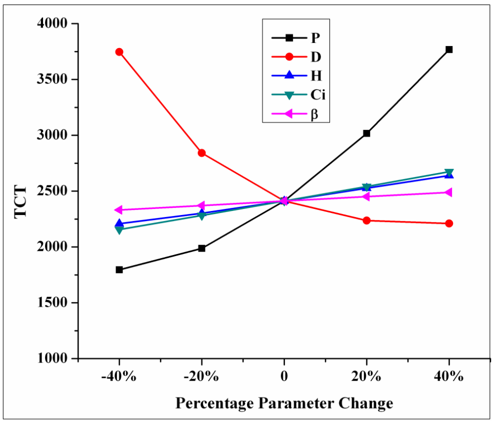


**Figure.3: TCT v/s Parameter Change**

**Table 2: Sensitivity Analysis of $T_2$ and $T_3$**

| Parameters | Parameter Changes | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | -40% | | -20% | | 20% | | 40% | |
| | $T_2$ | $T_3$ | $T_2$ | $T_3$ | $T_2$ | $T_3$ | $T_2$ | $T_3$ |
| P | 0.13 | 0.9 | 0.17 | 1.05 | 0.2 | 1.3 | 0.23 | 1.41 |
| D | 0.23 | 2 | 0.22 | 1.49 | 0.2 | 0.98 | 0.14 | 0.83 |
| β | 0.16 | 1.21 | 0.18 | 1.2 | 0.2 | 1.17 | 0.23 | 1.16 |
| H | 0.23 | 1.54 | 0.21 | 1.33 | 0.2 | 1.08 | 0.17 | 1.08 |
| Ci | 0.18 | 1.18 | 0.19 | 1.18 | 0.2 | 1.19 | 0.21 | 1.19 |

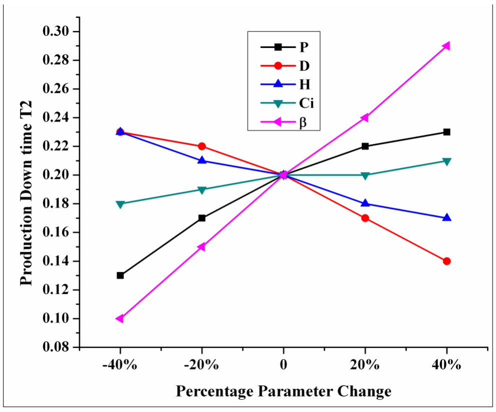


**Figure 4: $T_2$ v/s Parameter Change**

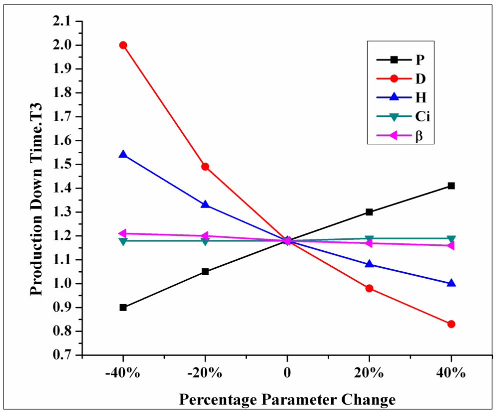


**Figure 5: $T_2$ v/s Parameter Change**

Values from table 2 and Figure 4 shows the sensitivity analysis of production down time $T_2$. Time $T_2$ is highly sensitive to production rate, demand, holding costs and rate of deterioration 'β'. An increase in the demand rate will result in a decrease in the length of downtime. This is expected since if demand rate is higher, the stock will take less time to finish. Increase in value of the rate of deterioration 'β' increases the inventory depletion time. With the deterioration rate higher, the stock will take more time to finish. Increase in deterioration rate follows linear relation with the increase in production time $T_2$. Though the rate of deterioration is basic during T2, but effect of the increase rate of deterioration is dominating. Increase in rate of production increases the inventory and hence required more time to finish. The inspection cost is also increasing function of down time.

Table 2, shows the sensitivity analysis of $T_3$. This time period is highly sensitive to demand rate, production rate and holding cost. During production down time demand used is a constant demand. Higher values of constant demand

gives lower values of production downtime. Increase in demand, decreases the down time and hence, optimum cycle time also. During $T_3$ rate of deterioration is assumed to be increased. Figure 5 indicates that, this increase rate of deterioration decreases the down time. The increased rate of deterioration reduces the inventory of good items and hence the time required to deplete this good item inventory decrease. This may led to loss to organization. As usual increased rate of production increases the time required to deplete build up inventory. The inspection cost is slightly sensitive to $T_3$.

## CONCLUSIONS

In this study, the theoretical EPQ model has been developed over an infinite time horizon. Some interesting observations are presented. During the inventory buildup time, demand used was inventory dependent and during an inventory depletion time it was constant Sensitivity analysis shows that TCT as well as production up time are highly sensitive to production and demand rate. Increase in rate of deteriorations increases the TCT and optimum cycle time. This model can be useful for the inventory managers in decision making, especially for the perishable items. The model can be further developed by considering different deterioration rate, production rate.

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